

Exotic Higgs boson decay modes as a harbinger of S_3 flavor symmetry

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Abstract

Discrete symmetries employed to explain flavor mixing and mass hierarchies can be associated with an enlarged scalar sector which might lead to exotic Higgs decay modes. In this paper, we explore such a possibility in a scenario with S_3 flavor symmetry which requires three scalar $SU(2)$ doublets. The spectrum is fixed by minimizing the scalar potential, and we observe that the symmetry of the model leads to tantalizing Higgs decay modes potentially observable at the CERN Large Hadron Collider (LHC).

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1 Introduction

Flavor models based on discrete symmetries are often used to address issues like large/small mixing and mild/strong mass hierarchies in the lepton/quark sector (for reviews, see, e.g. [1]). The permutation group S_3 is an attractive such candidate which was introduced in [2] and explored further in [3, 4]. In this paper we study the exciting prospect that such flavor models can predict *enlarged Higgs sectors with non-standard couplings to fermions and gauge bosons*, although such a nonsupersymmetric extension does not provide any additional stability to the potential. The main motivation here arises from flavor issues. Its supersymmetrization would indeed bring in quantum stability, but in this work we stick to a minimal nonsupersymmetric S_3 flavor scenario.

The motivation for choosing S_3 is that it is the smallest non-abelian discrete symmetry group that contains a 2-dimensional irreducible representation which can connect two maximally mixed generations. It is the symmetry group of an equilateral triangle and has three irreducible representations: $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$, with multiplication rules: $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{1}' + \mathbf{2}$ and $\mathbf{1}' \times \mathbf{1}' = \mathbf{1}$. Besides facilitating maximal mixing through its doublet representation, S_3 provides two inequivalent singlet representations which play a crucial role in reproducing fermion masses and mixing. To accomplish the latter, three scalar $SU(2)$ doublets are introduced, which couple to the fermions as dictated by S_3 symmetry. It so happens that large mixing among up- and down-type quarks cancel each other in the Cabibbo matrix. Neutrino masses are separately generated by a type-II see-saw mechanism using scalar $SU(2)$ triplets [5], so that the mismatch between the large mixing of the charged leptons and the diagonal neutrino masses translates directly into the Pontecorvo-Maki-Nakagawa-Sakata matrix. In this paper we do not deal with those triplets, but explore the following avenues: (i) minimization of the scalar potential with three scalar $SU(2)$ doublets, two of which form an S_3 doublet and the third an S_3 singlet, (ii) the gauge and Yukawa interactions of the neutral scalars, and (iii) different nonstandard production and decay modes of the neutral CP-even scalars leading to the possibility of their detection at the LHC.

For definiteness, we study the S_3 model pursued in [4] to explain the leptonic flavor structure. We concentrate on the complementary aspects by exploring the scalar sector. The assignments of the fermion and scalar fields are as follows:

$$\begin{aligned}
 (L_\mu, L_\tau) &\in \mathbf{2} & L_e, e^c, \mu^c &\in \mathbf{1} & \tau^c &\in \mathbf{1}', \\
 (Q_2, Q_3) &\in \mathbf{2} & Q_1, u^c, c^c, d^c, s^c &\in \mathbf{1} & b^c, t^c &\in \mathbf{1}', \\
 (\phi_1, \phi_2) &\in \mathbf{2} & \phi_3 &\in \mathbf{1},
 \end{aligned} \tag{1}$$

where the notations are standard and self-explanatory. The vacuum expectation values (VEVs) of the three scalar doublets $\phi_{1,2,3}$ induce spontaneous electroweak symmetry breaking (SSB).

2 Scalar potential and spectrum

The most general S_3 invariant scalar potential involving three scalar doublet fields is given by [4, 6]

$$V = m^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + m_3^2\phi_3^\dagger\phi_3 + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 + \frac{\lambda_2}{2}(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2 + \lambda_3\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1 + \frac{\lambda_4}{2}(\phi_3^\dagger\phi_3)^2 \\ + \lambda_5(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \lambda_6\phi_3^\dagger(\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger)\phi_3 + \left[\lambda_7\phi_3^\dagger\phi_1\phi_3^\dagger\phi_2 + \lambda_8\phi_3^\dagger(\phi_1\phi_2^\dagger\phi_1 + \phi_2\phi_1^\dagger\phi_2) + \text{h. c.} \right]. \quad (2)$$

After SSB, nine degrees of freedom are left: three neutral scalars, two neutral pseudoscalars and two charged scalars with two degrees of freedom each. We denote the VEVs of ϕ_i by v_i and assume the λ_i 's to be real. For the purpose of generating maximal mixing in the lepton sector, we choose the vacuum alignment $v_1 = v_2 = v$. Once we solve the tadpole equations, $v_1 = v_2$ turns out to be an extremal condition if the following relations are satisfied:

$$-m^2 = (2\lambda_1 + \lambda_3)v^2 + (\lambda_5 + \lambda_6 + \lambda_7)v_3^2 + 3\lambda_8vv_3, \\ -m_3^2 = \lambda_4v_3^2 + 2(\lambda_5 + \lambda_6 + \lambda_7)v^2 + 2\lambda_8v^3/v_3. \quad (3)$$

To ensure that the chosen vacuum alignment actually corresponds to a minimum of the potential, we adjust parameters to make sure that the determinant of the Hessian matrix is positive. Just to recall, the Hessian is defined as the square matrix of second order partial derivatives of a function describing its local curvatures. In this case, the function is the scalar potential and the Hessian is just the mass matrix of the scalars. The positivity of the eigenvalues – see later in Eq. (6) – guarantees that the potential is minimized. We note that after SSB, the potential turns out to be a polynomial of order four, and its global stability in the asymptotic limit (i.e. $\phi_i \rightarrow \infty$) is ensured by the following set of conditions:

$$\lambda_1 + \lambda_2 > 0, \quad \lambda_1 + \lambda_3 > \lambda_2, \quad \lambda_4 > 0, \quad \lambda_5 + \lambda_6 > 0, \quad \lambda_7 > 0, \quad \lambda_8 > 0. \quad (4)$$

We now set out to find the spectrum of the three CP-even neutral scalars. We insert the expansion $\phi_i^0 = v_i + h_i$ in Eq. (2) to obtain the mass matrix. After its diagonalization the weak basis scalars $h_{1,2,3}$ are expressed in terms of the physical scalars $h_{a,b,c}$ as

$$h_1 = U_{1b} h_b + U_{1c} h_c - \frac{1}{\sqrt{2}} h_a, \\ h_2 = U_{2b} h_b + U_{2c} h_c + \frac{1}{\sqrt{2}} h_a, \\ h_3 = U_{3b} h_b + U_{3c} h_c, \quad (5)$$

where U_{ib} and U_{ic} are analytically tractable but complicated functions of λ_i s, v and v_3 , which we do not display. The condition $v_1 = v_2$ immediately leads to $U_{1b} = U_{2b}$ and $U_{1c} = U_{2c}$. The masses of the three CP-even neutral scalars are

$$m_a^2 = 4\lambda_2v^2 - 2\lambda_3v^2 - v_3(2\lambda_7v_3 + 5\lambda_8v), \\ m_{b(c)}^2 = \frac{1}{2v_3} [4\lambda_1v^2v_3 + 2\lambda_3v^2v_3 + 2\lambda_4v_3^3 - 2\lambda_8v^3 + 3\lambda_8vv_3^2 \mp \Delta m^3]; \quad (6)$$

where

$$\Delta m^3 = \left[8vv_3 \left\{ 2vv_3^3 \left(2(\lambda_5 + \lambda_6 + \lambda_7)^2 - \lambda_4(2\lambda_1 + \lambda_3) \right) + 2\lambda_8v^4(2\lambda_1 + \lambda_3) - 3\lambda_4\lambda_8v_3^4 \right. \right. \\ \left. \left. + 12\lambda_8v^2v_3^2(\lambda_5 + \lambda_6 + \lambda_7) + 12\lambda_8^2v^3v_3 \right\} + \left\{ 2v^2v_3(2\lambda_1 + \lambda_3) + 2\lambda_4v_3^3 - 2\lambda_8v^3 + 3\lambda_8vv_3^2 \right\}^2 \right]^{\frac{1}{2}}. \quad (7)$$

A few things are worth noting at this stage:

- (i) Since $\phi_{1,2,3}$ are all weak $SU(2)$ doublets, their VEVs are related as: $2v^2 + v_3^2 = v_{\text{SM}}^2$, where $v_{\text{SM}} \approx 246$ GeV.
- (ii) One of the physical scalars is given by $h_a = (h_2 - h_1)/\sqrt{2}$, i.e. there is no dependence on $\lambda_{\{1,\dots,8\}}$ or on the VEVs. This happens because S_3 symmetry requires the scalar mass matrix to be of the form

$$\begin{pmatrix} a & b & c \\ b & a & c \\ c & c & d \end{pmatrix},$$

which always yields $(-1, 1, 0)$ as one eigenvector, regardless of the values of a, b, c and d .

- (iii) We strictly follow Eq. (4) to ensure that the potential is bounded from below. We randomly vary the *magnitude* of the λ_i 's in the range $[0, 1]$, although slightly larger (but $< 4\pi$) values of $|\lambda_i|$ would have still kept the couplings perturbative. We *accept* a given set of $\{\lambda_1, \dots, \lambda_8, v\}$ only if it satisfies the minimization conditions.
- (iv) The difference $m_c^2 - m_b^2 = \Delta m^3/v_3$ is positive, and when $v_3 \rightarrow 0$, i.e. $v \rightarrow v_{\text{SM}}$, the splitting grows enormously. Since the maximum value of m_c^2 is controlled by $\lambda_i \leq 1$, m_b^2 becomes tachyonic when $v_3 \rightarrow 0$. It has been suggested in [4] that with *order one* Yukawa couplings, the ratio $v_3/v \sim 0.1$ reproduces the correct Cabibbo angle in the quark sector. We require $v_3/v \geq 0.6$ to ensure that m_b^2 stays above the accepted limit. Since h_b and h_c have similar gauge and Yukawa properties, quite different from those of h_a (see discussions later), we show the mass splitting ($m_c - m_b$) against m_b in Fig. 1(a), and the relation between m_b and m_a in Fig. 1(b).

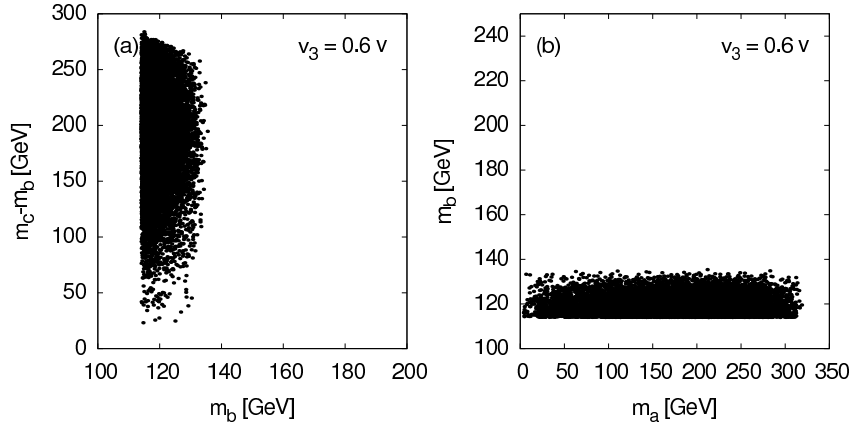


Figure 1: Results of a random search for allowed scalar masses for a fixed $v_3/v = 0.6$. In the left panel (a), we exhibit the splitting ($m_c - m_b$) for different choices of m_b . In the right panel (b), we show the allowed range of m_a .

3 Scalar couplings to gauge and matter fields

The kinetic terms $|D_\mu \phi_i|^2$ ($i = 1, 2, 3$) yield the couplings of the symmetry basis h_i to W^\pm and Z . Clearly, these couplings are modified by a factor of $v_i/v_{\text{SM}} < 1$ compared to their SM expressions. In terms of the mass basis scalars, we observe the following: (i) The coupling of h_b to W^+W^- (or, ZZ) is the corresponding SM coupling multiplied by $(2vU_{1b} + v_3U_{3b})/v_{\text{SM}}$ and the corresponding factor for h_c is $(2vU_{1c} + v_3U_{3c})/v_{\text{SM}}$. (ii) The scalar h_a does not have $h_a ZZ$ or $h_a WW$ couplings, unlike the other two scalars. This can be understood as follows. The gauge couplings of h_i arise from the linear expansion $\phi_i^0 = v_i + h_i$ in the kinetic term. Since $v_1 = v_2 = v$, the combination $(h_1 + h_2)$ will couple to gauge bosons as proportional to v . The orthogonal combination $(h_2 - h_1)$ that represents the physical scalar h_a – see Eq. (5) and point ii following Eq. (7) – will not have the usual scalar-gauge-gauge vertex. The four-point $h_a^2 ZZ$ and $h_a^2 WW$ couplings will, however, exist.

The S_3 invariant Yukawa Lagrangian, where the neutral scalars and the charged leptons/quarks are in their weak basis, is given by (f_i 's are the leptonic and $g_i^{u/d}$'s are quark Yukawa couplings)

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} = & f_4 e e^c h_3 + f_5 e \mu^c h_3 + f_1 \mu^c (\mu h_2 + \tau h_1) + f_2 \tau^c (-\mu h_2 + \tau h_1) \\ & + g_4^u u u^c h_3 + g_5^u u c^c h_3 + g_1^u c^c (c h_2 + t h_1) + g_2^{u^c} (-c h_2 + t h_1) \\ & + g_4^d d d^c h_3 + g_5^d d s^c h_3 + g_1^d s^c (s h_2 + b h_1) + g_2^{d^c} (-s h_2 + b h_1) + \text{H. c.}\end{aligned}\quad (8)$$

The couplings of $h_{b,c}$ to the quarks and leptons depend on the parameters v, v_3, λ_i and f_i (or, $g_i^{u/d}$), while the couplings of h_a to fermions depend only on f_i (or, $g_i^{u/d}$). The *physical* scalar couplings to the *mass basis* fermions are given by the following Yukawa matrices, displayed for the charged leptons as an example (the structures for the quark sector are similar *modulo* Cabibbo mixing):

$$Y_{h_a} = \begin{pmatrix} 0 & 0 & Y_{e_L \tau_R}^a \\ 0 & 0 & Y_{\mu_L \tau_R}^a \\ Y_{\tau_L e_R}^a & Y_{\tau_L \mu_R}^a & 0 \end{pmatrix}, \quad Y_{h_{b,c}} = \begin{pmatrix} Y_{e_L e_R}^{b,c} & Y_{e_L \mu_R}^{b,c} & 0 \\ Y_{\mu_L e_R}^{b,c} & Y_{\mu_L \mu_R}^{b,c} & 0 \\ 0 & 0 & Y_{\tau_L \tau_R}^{b,c} \end{pmatrix}. \quad (9)$$

The position of the zeros in the matrices deserves some attention. It turns out that $h_{a,b,c}$ have off-diagonal fermion couplings at tree level due to the absence of any natural flavor conservation [7]. The numerical entries of the Yukawa matrices are intimately tied to the successful reproduction of the quarks' and leptons' masses and mixings. We make three observations at this stage. (i) We admit that unless some S_3 breaking parameters are introduced a successful reproduction of V_{cb} and V_{ts} is problematic [8], and also domain walls will be formed. We do not intend to cover all flavor issues. We simply concentrate on the scalar sector whose Lagrangian is S_3 invariant to start with. (ii) h_a couples *only* off-diagonally and one of the two fermions has to be from the third generation. (iii) $h_{b,c}$ couple diagonally as in the SM, *but also* possess small, numerically insignificant, off-diagonal couplings involving the first two generations.

The last two points require further clarification. In a theory with more than one $SU(2)$ Higgs doublet, tree level flavor changing neutral currents (FCNC) generally exist in the scalar sector. For example, they exist in the ordinary 'two Higgs doublet model (2HDM)', but in the supersymmetric standard model they are avoided by the arrangement that one doublet couples only to the up-type fermions and the other to only the down-types. In nonsupersymmetric scenarios, in the absence of any natural flavor conservation, symmetry arguments have been advanced in the context of multi-Higgs models to show that the off-diagonal Yukawa couplings of the neutral scalars are suppressed by their relation to the off-diagonal entries of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [9].

In the present case, S_3 symmetry, under which both scalars and fermions transform nontrivially, is instrumental in suppressing the off-diagonal couplings. To provide intuitive understanding, we take, as an example, only the two-flavor μ - τ sector together with two neutral scalars h_1 and h_2 . It is not difficult to see that the combination $(h_2 - h_1)$, which corresponds to h_a , couples only off-diagonally, as mentioned earlier. But the other combination $(h_2 + h_1)$, which corresponds to $h_{b,c}$ following Eq. (5), couples only diagonally to physical μ or τ . When we consider the quark sector, μ and τ would be replaced by second and third generation quarks which will have CKM mixing. This will yield off-diagonal entries for $h_{b,c}$ couplings to quarks suppressed by the off-diagonal CKM elements. The same happens for off-diagonal couplings involving the first two generations as well. The tiny size of tree level FCNC rates in an S_3 flavor model has been noticed also earlier, where predictions for $\text{Br}(\tau \rightarrow 3\mu)$, $\text{Br}(K_L \rightarrow 2e)$ and $\text{Br}(B_s \rightarrow 2\mu)$ have been given [10]. In some setups where the fermion transformations under S_3 are not appropriately adjusted, the off-diagonal Yukawa couplings may become order one which induce sizable neutral scalar mediated rare processes, like $K_L \rightarrow \mu e$ or $K_L \rightarrow 2\pi$, at tree level. This requires those neutral scalars to lie beyond several TeV [6, 11]. But in our case, once we adjust the $f_i/g_i^{u,d}$'s to reproduce the fermion masses and mixing, the off-diagonal Yukawa couplings are determined too. The largest of them corresponds to $\bar{c}_L t_R h_a$, which is about 0.8. The second largest off-diagonal coupling is that for $\bar{s}_L b_R h_a$, and is about 0.02. The next in line is $\bar{\mu}_L \tau_R h_a$, whose coefficient is about 0.008. The others are orders of magnitude smaller, and are of no numerical significance. Although FCNC processes like $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings proceed at tree level, the contributions are adequately suppressed even for light scalar mediators.

4 Collider signatures

The perturbativity condition $|\lambda_i| \leq 1$ and the requirement $m_{b/c} \geq 114 \text{ GeV}$ (for which we set $v_3/v \simeq 0.6$) yields m_b in the neighbourhood of 120 GeV and m_c within 400 GeV – see the scatter plots in Fig. 1. Both h_b and h_c would decay into the *usual* ZZ , WW , $b\bar{b}$, $\gamma\gamma$, \dots modes, but the dominant decay mode of h_b (or h_c) for the case of $m_a < m_b/2$ (or $m_a < m_c/2$) would be into $h_a h_a$. Recall that the existing limits on the Higgs mass depend crucially on the gauge coupling of the Higgs. Since $h_a ZZ$ or $h_a WW$ couplings are nonexistent, the mass of h_a is unconstrained, i.e. m_a can be lower than 114 GeV or larger than 200 GeV. We numerically calculate the strength of the $h_b h_a h_a$ coupling from the set of *acceptable* parameters characterizing the potential, and introduce a parameter k which is the ratio of the $h_b h_a h_a$ coupling and the $h_b WW$ coupling. The magnitude of k depends on the choice of λ_i and v_3 . Assuming $m_a = 50 \text{ GeV}$, we obtain k in the range of $(5 - 30)$. Just to compare with a 2HDM [12] for illustration, the corresponding k value, when the heavier Higgs weighing around 400 GeV decays into two lighter Higgs weighing 114 GeV each, is about 10.

In Fig. 2(a) we have plotted the branching ratio of $h_b \rightarrow h_a h_a$ as a function of m_b for two representative values $m_a = 50, 75 \text{ GeV}$, and for $k \sim 5$ and 30, which correspond to the smallest and largest k obtained from the set of *accepted* scalar parameters. We observe that till the WW or ZZ decay modes open up, the branching ratio $h_b \rightarrow h_a h_a$ is almost 100%. To calculate the decay widths into the usual modes (other than $h_a h_a$), we have used HDECAY [13] by appropriately modifying the gauge and Yukawa couplings.

As Fig. 2(b) suggests, as long as $m_a < m_t$, h_a will dominantly decay into jets, and one of them can be identified as the b -jet. The branching ratio of $h_a \rightarrow \mu\bar{\tau}$ is, nevertheless, not negligible (about 0.1). As shown in Fig. 2(c), for $m_a \ll m_t$, the branching ratio of $t \rightarrow h_a c$ is quite sizable, which falls with increasing m_a . It may be possible to reconstruct h_a from $h_a \rightarrow \mu\bar{\tau}$. In fact, a light h_a would be copiously produced from the top decay at the LHC. On the other hand, if $m_a > m_t$, as can be seen again from Fig. 2(b), h_a decays to $t\bar{c}$ with an almost 100% branching ratio.

If k is large, then there is an interesting twist to the failed Higgs search at LEP-2. In this case, $h_b \rightarrow h_a h_a$ would overwhelm $h_b \rightarrow b\bar{b}$, and hence the conventional search for the SM-like scalar (h_b , as the lighter between h_b and h_c) would fail. This is similar to what happens in the next-to-minimal supersymmetric models, when the lightest scalar would dominantly decay into two pseudoscalars, and each pseudoscalar would then decay into $2b$ or 2τ final states. In view of these possible $4b$ or 4τ Higgs signals, LEP data have been reanalyzed putting constraints on the Higgs production cross section times the decay branching ratios [14, 15]. The possibility of the Higgs cascade decays into $4j$ ($j = \text{quark/gluon}$), $2j + 2$ photons and 4 photons has been studied too [16, 17]. From a study of $4b$ final states, a limit $m_h > 110 \text{ GeV}$ (for a SM-like Higgs) has been obtained [16]. From all other cascade decays the limit on m_h will be considerably weaker. Our h_a has the special feature that it has only off-diagonal Yukawa couplings involving one third-family fermion. If h_b is lighter than the top quark, it would decay as $h_b \rightarrow h_a h_a \rightarrow 2b + 2j$, and into $b + 1j + \mu + \tau$, the latter constituting a spectacular signal with two different lepton flavors μ and τ . The standard $2b$ and cascade $4b$ decay searches are not sensitive to our final states, and so a value of m_b much lighter than 110 GeV is not ruled out.

5 Conclusions

The discrete flavor symmetry S_3 , besides successfully reproducing fermion masses and mixing, provides an extended Higgs sector having unconventional decay properties. We assume all the couplings to be real, and do not deal with the possibility of CP violation in this paper. The potential has been minimized requiring maximal mixing for the atmospheric neutrinos. In our setup, there are two scalars which are SM Higgs like, *except* that each of them can have a dominant decay into the third ($h_{b,c} \rightarrow h_a h_a$). The latter, i.e. h_a , has no $h_a VV$ -type gauge interactions, and has *only* flavor off-diagonal Yukawa couplings with one fermion from the third generation. It is not unlikely that by evading the conventional search strategies, both h_b and h_a are already buried in the existing LEP and Tevatron data. In this analysis we have not dealt with the two pseudoscalars, which we leave for a future study. We urge our experimental colleagues to look for our suggested signals at the LHC, and perhaps also reanalyze the existing collider data.

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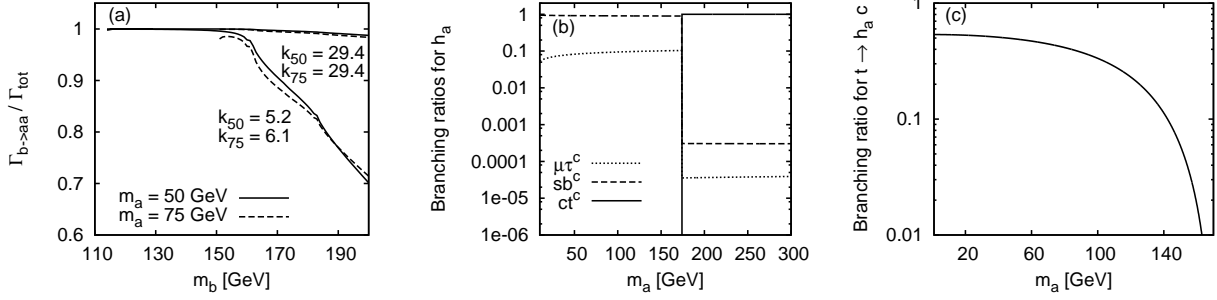


Figure 2: (a) Branching ratio of $h_b \rightarrow h_a h_a$ for two representative values of m_a , and in each case for smallest / largest values of k in the set of accepted scalar parameters which compares the strength of the $h_b h_a h_a$ coupling to the strength of $h_b W W$ coupling; (b) branching ratios for the decay of h_a , and (c) branching ratio of the top quark decay into h_a and charm quark.

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